Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

JOURNAL OF SCIENTIFIC LETTERS
www.jslsci.com

GEOMETRICAL ANALYSIS OF BLACK HOLE SHADOWS IN DIFFERENT GRAVITATIONAL SPACETIMES

Komal Prakash Kaitake
Research Scholar, Department of Physics, Jayoti Vidyapeeth Women's University, Jaipur, Rajasthan
Dr. Sumit
Research Supervisor, Department of Physics, Jayoti Vidyapeeth Women's University, Jaipur, Rajasthan

ABSTRACT

Black hole shadows provide significant insights into spacetime geometry and the behaviour of light under extreme gravitational conditions. The present study examines the geometrical characteristics of black hole shadows in different gravitational spacetimes, focusing on their size, shape, symmetry, and boundary structures. Particular attention is given to Schwarzschild, Kerr, Reissner–Nordström, and Kerr–Newman black hole models. The investigation explores how mass, angular momentum, electric charge, and gravitational modifications influence photon trajectories and shadow formation. It also considers the relationship between photon spheres, gravitational lensing, and the apparent geometry of black hole shadows. A comparative analytical approach is adopted to understand the differences between various gravitational configurations. The study aims to establish a comprehensive theoretical understanding of shadow geometry and demonstrate its importance in investigating strong gravitational fields and testing the predictions of general relativity.

Keywords: Black holes, Shadow geometry, Gravitational spacetime, Photon sphere, General relativity, Gravitational lensing, Kerr black hole.

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

I. INTRODUCTION

Black holes are among the most remarkable objects predicted by Einstein's general theory of relativity. They represent regions of spacetime where gravitational attraction becomes extremely strong, preventing matter and radiation from escaping beyond the event horizon. Their existence produces significant changes in the surrounding gravitational environment and influences the movement of nearby particles and photons. One of the most important features associated with black holes is their shadow, which appears as a dark region against the surrounding luminous background. This phenomenon results from the strong gravitational bending and capture of light rays. The geometrical characteristics of black hole shadows provide valuable information about the physical properties of black holes and the structure of spacetime.

Different gravitational spacetimes produce different shadow characteristics depending on their physical parameters and geometrical structures. The Schwarzschild model describes a non-rotating and uncharged black hole and produces a circular shadow under ideal observational conditions. The Kerr model represents a rotating black hole, in which frame dragging influences photon trajectories and produces characteristic changes in shadow appearance. Similarly, the Reissner–Nordström model incorporates electric charge, which modifies the gravitational field and affects the location of photon orbits. The Kerr–Newman model combines rotation and electric charge, creating a more complex gravitational configuration. These models provide important theoretical opportunities for examining the relationship between gravitational properties and shadow geometry.

The formation of black hole shadows is closely associated with the behaviour of photons moving through curved spacetime. Under strong gravitational conditions, light follows curved trajectories rather than travelling along straight paths. Certain photons may approach unstable circular or spherical orbits before escaping or falling into the black hole. These critical trajectories establish the boundary of the shadow as perceived by a distant observer. The location of photon regions, the gravitational influence of the black hole, and the observer's position collectively determine the apparent shadow structure. Consequently, investigating photon motion is essential for understanding the geometrical characteristics of black hole shadows.

The present study focuses on the geometrical analysis of black hole shadows in different gravitational spacetimes. It examines the influence of mass, angular momentum, electric charge, and other gravitational parameters on shadow size, shape, symmetry, and deformation. Particular

E-mail: editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

attention is given to comparing static, rotating, and charged black hole configurations. The investigation also considers the importance of photon trajectories and gravitational lensing in determining shadow boundaries. Through a comparative analytical approach, the study aims to improve the theoretical understanding of black hole geometry and explore the potential significance of shadow characteristics in testing general relativity and alternative gravitational theories.

II. THEORETICAL BACKGROUND OF BLACK HOLE SHADOW GEOMETRY

Black hole shadow geometry is fundamentally connected with the structure of spacetime surrounding a gravitational object. According to general relativity, massive objects produce curvature in spacetime, influencing the movement of matter and radiation. A black hole creates an exceptionally strong gravitational environment in which the trajectories of nearby photons are significantly modified. The shadow represents the dark region associated with light rays that are captured by the black hole rather than reaching the observer. Its boundary is determined by critical photon trajectories and therefore provides information about the gravitational structure surrounding the event horizon. The study of shadow geometry requires consideration of the spacetime configuration, photon motion, and observational perspective.

The Schwarzschild black hole provides the simplest theoretical model for examining shadow geometry. It describes a static, spherically symmetric gravitational object without electric charge. Because of its spherical symmetry, the photon trajectories exhibit equivalent behaviour in different angular directions. Consequently, the apparent shadow is circular for a distant observer under ideal conditions. The location of the unstable photon sphere determines the critical region associated with the shadow boundary. The resulting circular structure provides an important reference for investigating more complicated black hole configurations. Comparisons with this basic model help identify geometrical changes introduced by rotation, charge, and other gravitational properties.

The Kerr black hole represents a rotating gravitational configuration and introduces additional complexity into shadow formation. Rotation influences the surrounding spacetime through frame dragging, which changes the trajectories of photons moving near the black hole. Photons travelling in the direction of rotation experience different orbital conditions from those travelling in the opposite direction. These differences influence the apparent dimensions and symmetry of the shadow. The observer's inclination relative to the rotational axis also affects its appearance.

E-mail: editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

Therefore, the geometrical structure of a Kerr black hole shadow depends on both intrinsic rotational properties and observational orientation.

The Reissner–Nordström black hole introduces electric charge as an additional physical parameter. Its gravitational structure differs from the Schwarzschild configuration because electromagnetic effects influence the surrounding spacetime geometry. The presence of charge modifies the location of critical photon trajectories and may alter the apparent shadow radius. Although the solution remains spherically symmetric, its shadow dimensions can differ from those of an uncharged black hole having the same mass. The Kerr–Newman configuration extends this analysis by combining electric charge with angular momentum. It provides an opportunity to investigate the combined influence of rotational and electromagnetic properties on shadow geometry.

Other theoretical gravitational configurations may also produce variations in black hole shadow characteristics. Modified gravity models, cosmological black hole solutions, and configurations involving additional gravitational fields can introduce changes in the spacetime structure. Such modifications may influence photon trajectories, the location of critical orbits, and the apparent boundary of the shadow. The resulting geometrical differences are important for examining the predictions of general relativity and alternative theories. However, the interpretation of these differences requires careful consideration of the assumptions and physical parameters associated with each model.

The theoretical background of black hole shadow geometry therefore involves the combined study of spacetime structure, photon motion, gravitational lensing, and observer-dependent effects. Static and spherical configurations provide simple reference models, whereas rotating and charged configurations demonstrate more complicated geometrical behaviour. Modified gravitational models further expand the range of possible shadow structures. A systematic understanding of these configurations is necessary for comparing theoretical predictions and identifying the physical characteristics responsible for variations in black hole shadows.

III. GEOMETRICAL CHARACTERISTICS OF DIFFERENT GRAVITATIONAL SPACETIMES

The geometrical characteristics of black hole shadows vary according to the physical properties of the central gravitational object. Size, circularity, symmetry, boundary structure, and apparent displacement are among the important features used to describe shadow morphology. These

E-mail: editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

characteristics are closely related to the gravitational field surrounding the black hole and the movement of photons near its critical region. Different gravitational solutions provide distinct theoretical environments for examining these features. A comparative investigation of these configurations helps explain how physical parameters influence the apparent geometry of black hole shadows.

The Schwarzschild black hole provides a fundamental reference for studying the geometrical properties of shadows. Its spherical symmetry produces a circular shadow for a distant observer under ideal conditions. The shadow boundary is associated with unstable photon trajectories surrounding the black hole. Since the gravitational field is independent of any rotational direction, the shadow does not exhibit rotational displacement or asymmetry. Its characteristic size is primarily determined by the mass of the black hole and the distance of the observer. This simple configuration is useful for understanding the relationship between gravitational strength and apparent shadow dimensions.

In contrast, the Kerr black hole exhibits a more complex shadow structure because of its angular momentum. The rotation of the black hole produces frame dragging, which influences photons moving near the gravitational object. Prograde and retrograde photon trajectories experience different gravitational conditions, resulting in variations in the apparent shadow boundary. The shadow may become asymmetric and displaced relative to the centre of the projected image. Its geometrical appearance also depends on the observer's inclination. A viewing position close to the rotational axis may produce a relatively symmetric appearance, whereas an equatorial viewing position may reveal more pronounced deformation.

The Reissner–Nordström black hole introduces electric charge into the gravitational configuration. The presence of charge influences the gravitational field and changes the conditions governing photon motion. As a result, the critical photon region and apparent shadow dimensions may differ from those associated with the Schwarzschild model. However, because the configuration remains spherically symmetric, its shadow retains a circular appearance for a distant observer. This distinction demonstrates that shadow size and shadow shape are separate geometrical characteristics. A shadow may remain circular while its overall dimensions change because of variations in the underlying gravitational parameters.

The Kerr–Newman black hole combines the effects of rotation and electric charge. Its shadow geometry is influenced by both parameters, producing a more complicated relationship between

E-mail: editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

physical properties and observable characteristics. Rotation may introduce asymmetry and displacement, while electric charge may modify the critical photon region. The combined influence of these parameters can change the apparent size and boundary structure. Such configurations are important for understanding how multiple gravitational properties interact to determine the appearance of black hole shadows. They also provide a useful theoretical framework for investigating charged rotating black holes.

Modified gravitational theories introduce additional possibilities for examining shadow geometry. Some models incorporate alternative gravitational fields, higher-dimensional structures, or corrections to the standard description of gravity. These modifications may influence the location of photon orbits and the overall appearance of the shadow. Depending on the theoretical model, the resulting shadow may display changes in size, circularity, or boundary characteristics. Comparative studies of these configurations may help identify geometrical features that distinguish general relativity from alternative theories. Nevertheless, the physical interpretation of such differences requires detailed theoretical analysis and appropriate observational constraints.

IV. PHOTON TRAJECTORIES, GRAVITATIONAL LENSING AND SHADOW FORMATION

Photon trajectories play a fundamental role in determining the appearance and boundary of black hole shadows. In ordinary conditions, light travels approximately along straight paths, but strong gravitational fields significantly modify its direction of propagation. Near a black hole, photons follow curved trajectories determined by spacetime geometry. Some photons may escape towards distant observers, while others are captured by the black hole. The boundary between these two groups of trajectories determines the apparent shadow. Therefore, the investigation of photon motion is essential for understanding how black hole shadows form and why their geometrical characteristics vary between different gravitational configurations.

The photon sphere is particularly important in the formation of shadows associated with static and spherically symmetric black holes. It represents a region where photons can follow unstable circular trajectories around the gravitational object. These orbits are unstable because small changes in the photon trajectory may result in escape or capture. Photons approaching the critical region may undergo substantial gravitational bending before reaching an observer. The photon sphere therefore establishes an important relationship between the gravitational structure of a black hole and the apparent dimensions of its shadow. In rotating configurations, the

E-mail: editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

corresponding photon region becomes more complicated because different spherical photon orbits are possible.

Gravitational lensing further influences the appearance of black hole shadows. When light passes near a black hole, the curvature of spacetime changes its propagation direction. This effect may produce highly distorted images of surrounding matter and radiation. Light emitted behind or near a black hole can be bent around the gravitational object and reach the observer from different directions. Such processes contribute to the bright emission structures surrounding the dark shadow. The strength of gravitational lensing depends on the gravitational field, photon trajectory, and location of the emitting material. Consequently, lensing must be considered when interpreting the apparent boundary and surrounding brightness distribution of a black hole image.

The characteristics of photon trajectories differ significantly between static and rotating gravitational spacetimes. In Schwarzschild geometry, the spherical symmetry produces a relatively simple photon orbit structure. In Kerr geometry, frame dragging influences the direction and location of photon trajectories. The resulting photon region is more complex and depends on the black hole's angular momentum and observer orientation. Charged configurations introduce further modifications to the gravitational environment. These differences demonstrate that photon trajectories are directly connected with the geometrical properties of the underlying spacetime. Comparative analysis of these trajectories can therefore reveal the physical mechanisms responsible for variations in shadow structure.

Observer position is another important factor in determining the apparent geometry of a black hole shadow. A distant observer and an observer located closer to the gravitational object may perceive different angular characteristics. The inclination of the observer relative to the rotational axis can also influence the apparent symmetry of a rotating black hole shadow. Similarly, the presence of surrounding matter and radiation may affect the visibility and brightness distribution of the shadow. These observational factors must be considered when comparing theoretical models. An accurate description of shadow geometry therefore requires a combined understanding of spacetime structure, photon motion, gravitational lensing, and observer-dependent effects.

The study of photon trajectories and gravitational lensing provides a comprehensive foundation for investigating black hole shadow formation. Critical photon orbits determine the transition between captured and escaping light rays, while gravitational lensing influences the apparent distribution of surrounding radiation. Differences in spacetime geometry modify both processes

E-mail:* *editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

and produce characteristic changes in shadow appearance. These relationships are important for developing theoretical comparisons between gravitational models. They also establish a foundation for connecting analytical investigations of black hole shadows with modern astronomical observations.

V. CONCLUSION

The geometrical analysis of black hole shadows provides an important approach for understanding the structure of spacetime and the behaviour of light under extreme gravitational conditions. The present study examines the characteristics of black hole shadows in different gravitational configurations, including Schwarzschild, Kerr, Reissner–Nordström, and Kerr–Newman models. The comparison demonstrates that shadow geometry is influenced by mass, angular momentum, electric charge, and observer orientation. Static spherical configurations generally produce circular shadows, whereas rotating configurations may exhibit asymmetry and displacement because of frame dragging.

The investigation also highlights the importance of photon trajectories and gravitational lensing in determining the apparent boundary of black hole shadows. Critical photon orbits establish the separation between escaping and captured light rays, while gravitational lensing influences the appearance of surrounding radiation. Differences in the gravitational structure modify the photon region and produce variations in shadow size and shape. These characteristics provide a useful theoretical basis for comparing different black hole configurations and understanding the relationship between gravitational properties and observable geometry.

The comparative analysis further demonstrates the significance of modified gravitational configurations in theoretical investigations. Alternative gravitational models may produce additional changes in shadow characteristics and provide opportunities for examining possible deviations from general relativity. However, these variations must be interpreted carefully because different physical parameters may produce similar geometrical appearances. Therefore, a comprehensive analysis involving multiple shadow characteristics and independent observational information is necessary for obtaining reliable conclusions about the underlying gravitational model.

Overall, the study establishes that black hole shadow geometry is an important subject in modern gravitational physics. The investigation of shadow size, circularity, symmetry, and boundary structure may contribute to a deeper understanding of strong gravitational fields. Future research

E-mail: editor.jslsci888@gmail.com

Online ISSN: 2456-1495
CODEN (USA): JSLOBJ

may incorporate advanced numerical simulations, realistic plasma environments, and improved astronomical observations. These developments may strengthen theoretical comparisons between different gravitational spacetimes and provide further opportunities for examining the predictions of general relativity and alternative theories of gravity.

REFERENCES

1. Synge, J. L. (1966). The escape of photons from gravitationally intense stars. *Monthly Notices of the Royal Astronomical Society, 131*(3), 463–466.

2. Bardeen, J. M. (1973). Timelike and null geodesics in the Kerr metric. In C. DeWitt & B. S. DeWitt (Eds.), *Black Holes* (pp. 215–239). Gordon and Breach.

3. Chandrasekhar, S. (1983). *The mathematical theory of black holes*. Oxford University Press.

4. Luminet, J. P. (1979). Image of a spherical black hole with thin accretion disk. *Astronomy and Astrophysics, 75*, 228–235.

5. Falcke, H., Melia, F., & Agol, E. (2000). Viewing the shadow of the black hole at the Galactic center. *The Astrophysical Journal Letters, 528*(1), L13–L16.

6. Grenzebach, A., Perlick, V., & Lämmerzahl, C. (2014). Photon regions and shadows of Kerr–Newman–NUT black holes with a cosmological constant. *Physical Review D, 89*, 124004.

7. Cunha, P. V. P., & Herdeiro, C. A. R. (2018). Shadows and strong gravitational lensing: A brief review. *General Relativity and Gravitation, 50*, 42.

8. Perlick, V., & Tsupko, O. Y. (2022). Calculating black hole shadows: Review of analytical studies. *Physics Reports, 947*, 1–39.

9. Qiao, C. K. (2022). Curvatures, photon spheres, and black hole shadows. *Physical Review D, 106*, 084060.

10. Takahashi, R. (2004). Shapes and positions of black hole shadows in accretion disks and spin parameters of black holes. *The Astrophysical Journal, 611*, 996–1004.

E-mail: editor.jslsci888@gmail.com